What is modulation?

- Modulation is the act of changing a carrier signal’s properties (amplitude, phase, frequency) in a controlled way in order to transmit data across a channel or to obtain desired signal properties.
Types of Digital Modulation

- Pulse Amplitude Modulation (PAM)
- Quadrature Amplitude Modulation (QAM)
- Phase Shift Keying (PSK)
- Frequency Modulation (FM)
Phase and Amplitude Modulation

- A phase and amplitude modulated carrier signal can be represented as:

\[ X(t) = a(t) \cos(2\pi f_c t + \theta(t)) \]

Where:
- \( a(t) \) is the amplitude modulation function
- \( \theta(t) \) is the phase modulation function
- \( f_c \) is the carrier frequency

- Which can be decomposed using the angle sum trigonometric identity into:

\[ X(t) = X_I(t) \cos(2\pi f_c t) - X_Q(t) \sin(2\pi f_c t) \]

Where:
- \( X_I(t) \) is the “in-phase” modulation function
- \( X_Q(t) \) is the “quadrature-phase” modulation function

This is a “bandpass” signal since the I and Q functions are modulating a carrier

Often called “I” and “Q”
Analytical Equivalent and Baseband Equivalent

- The “bandpass” signal can be decomposed into a “lowpass” modulation function multiplied by a complex exponential (carrier) by using Euler’s formula and defining a new “baseband equivalent” modulation function, $X_{bb}(t)$:

$$X_{bb}(t) = X_I(t) + jX_Q(t)$$

Then:

$$X(t) = X_I(t) \cos(2\pi f_c t) - X_Q(t) \sin(2\pi f_c t) = \text{Real}\{X_{bb}(t)e^{j2\pi f_c t}\}$$

Recall Euler’s formula:

$$e^{jx} = \cos x + j \sin x$$

Diagram:

- Baseband equivalent
- Analytic equivalent
- 0 Hz (DC) “Baseband” or “Lowpass”
- $f_c$ “RF” or “Bandpass”

Texas Instruments
Real Modulation Example (Real Mixing)

• As an example of real modulation, consider the case of mixing a low frequency sine wave to a higher frequency, such that \( X_{bb}(t) \) is simply a cosine wave:

\[
X_{bb}(t) = \cos(2\pi f_{IF} t)
\]

\[
= \frac{e^{j2\pi f_{IF} t} + e^{-j2\pi f_{IF} t}}{2}
\]

• The resulting modulated signal is:

\[
X(t) = \text{Real} \left\{ \left( \frac{e^{j2\pi f_{IF} t} + e^{-j2\pi f_{IF} t}}{2} \right) e^{j2\pi f_{ct}} \right\}
\]

\[
X(t) = \text{Real} \left\{ \frac{1}{2} e^{j2\pi f_{IF} t} e^{j2\pi f_{ct}} + \frac{1}{2} e^{-j2\pi f_{IF} t} e^{j2\pi f_{ct}} \right\}
\]

\[
X(t) = \text{Real} \left\{ \frac{1}{2} e^{j2\pi (f_c + f_{IF}) t} + e^{j2\pi (f_c - f_{IF}) t} \right\}
\]

\[
X(t) = \frac{1}{2} \cos(2\pi (f_c + f_{IF}) t) + \frac{1}{2} \cos(2\pi (f_c - f_{IF}) t)
\]

From Euler’s Undesired Image signal Desired signal Undesired Image signal
Complex Modulation Example (Complex Mixing)

- Define $X_{bb}(t)$ to be a complex exponential at the same frequency as the cosine in the previous example:

$$X_{bb}(t) = e^{j2\pi f_{IF}t} = \cos(2\pi f_{IF}t) + j\sin(2\pi f_{IF}t)$$

- The resulting modulated signal is:

$$X(t) = \text{Real}\{e^{j2\pi f_{IF}t}e^{j2\pi f_{ct}}\} = \text{Real}\{e^{j2\pi (f_c + f_{IF})t}\}$$

$$X(t) = \cos(2\pi (f_c + f_{IF})t)$$

From Euler’s

Desired signal only
Real vs Complex Example Visualized

Real signals have equivalent (mirrored) positive and negative frequency spectrums:

- Lower sideband
- Upper sideband

Complex signals have independent positive and negative frequency spectrums:

- Lower sideband
- Upper sideband

This type of transmission is called “double sideband” (DSB) transmission since the same information is transmitted in both sidebands.

This type of transmission is called “single sideband” (SSB) transmission since each sideband can transmit unique information.
Applications of Complex Modulation

- Image reject mixing uses the concept of complex modulation, exactly as shown in the complex modulation example, to reject the image signal in order to relax filtering requirements.

- Digital communications uses complex modulation to “double” the data rate for a given signal bandwidth, ultimately using sine and cosine as an orthogonal basis in order to transmit independently on the in-phase and quadrature-phase signals.

\[ X(t) = X_I(t) \cos(2\pi f_c t) - X_Q(t) \sin(2\pi f_c t) \]

Transmit independent information on both I and Q.
Digital Communications and Constellation Plots

- Constellation plots are used to visualize the complex baseband modulation function, $X_{bb}(t)$, by mapping the complex values onto the complex plane.

- Amplitude of the carrier is the distance from the origin:

\[ a(t) = \sqrt{X_I^2(t) + X_Q^2(t)} \]

- Phase of the carrier is the angle measured relative to the positive I axis:

\[ \theta(t) = \tan^{-1}\left(\frac{X_Q(t)}{X_I(t)}\right) \]
Find Transmitted Signal from Constellation Plot

• We want to transmit 0011 across the channel using the 16-QAM plot on the previous slide

• To send 0011, choose I = 1, Q = -3:

\[ X_{bb}(t) = X_I(t) + jX_Q(t) = 1 - j3 \]

• Carrier amplitude:

\[ a(t) = \sqrt{X_I^2(t) + X_Q^2(t)} = \sqrt{1^2 + (-3)^2} = \sqrt{10} \]

• Carrier phase:

\[ \theta(t) = \tan^{-1}\left(\frac{X_Q(t)}{X_I(t)}\right) = \tan^{-1}\left(-\frac{3}{1}\right) = -71.6^\circ = -1.25 \text{ rad} \]

• Resulting signal, X(t):

\[ X(t) = \sqrt{10} \cos(2\pi f_c t - 1.25) \]
\[ X(t) = \cos(2\pi f_c t) - 3 \sin(2\pi f_c t) \]

Easier to implement in digital hardware
Example Constellation Plots

More challenging noise and distortion requirements

Lower data rate (fewer bits per symbol)
Thanks for your time!