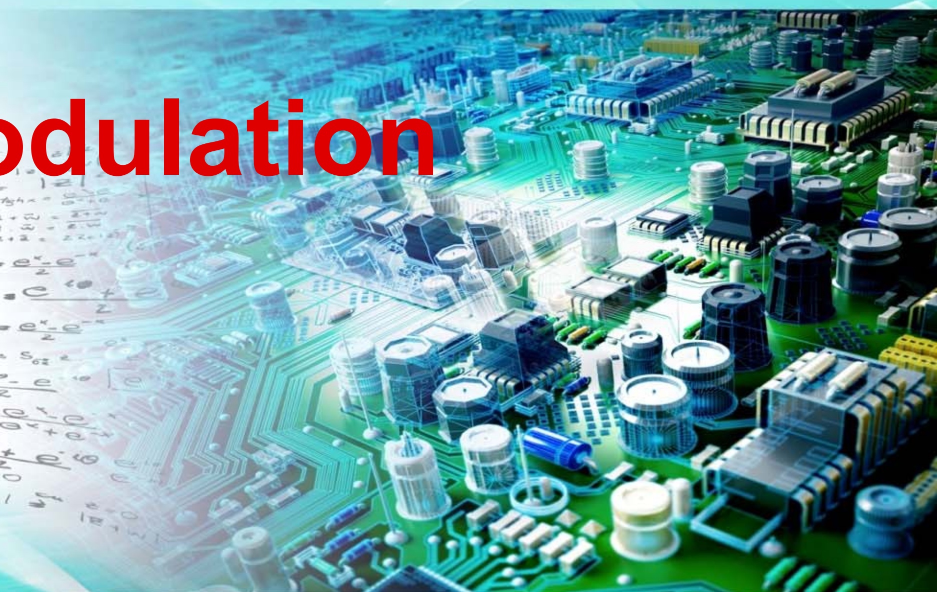
Real and Complex Modulation

TIPL 4708



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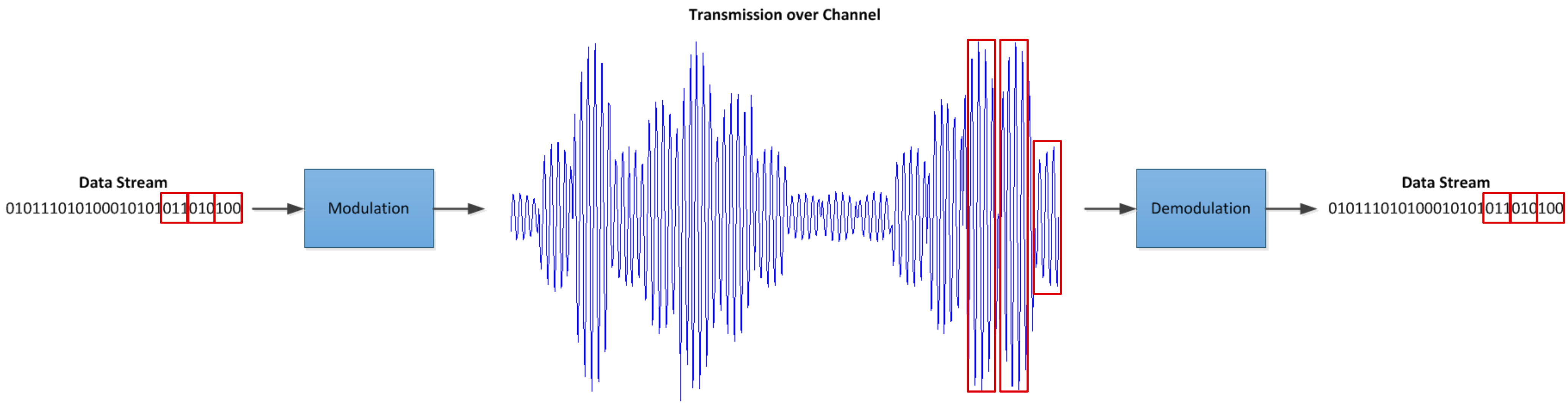
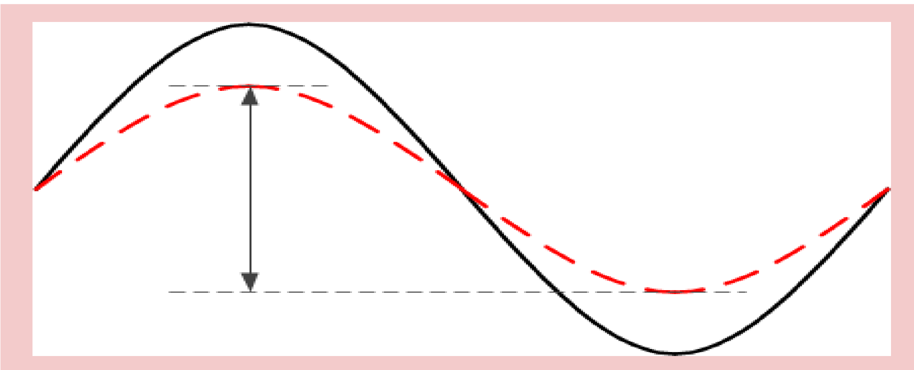
What is modulation?

- Modulation is the act of changing a carrier signal's properties (amplitude, phase, frequency) in a controlled way in order to transmit data across a channel or to obtain desired signal properties

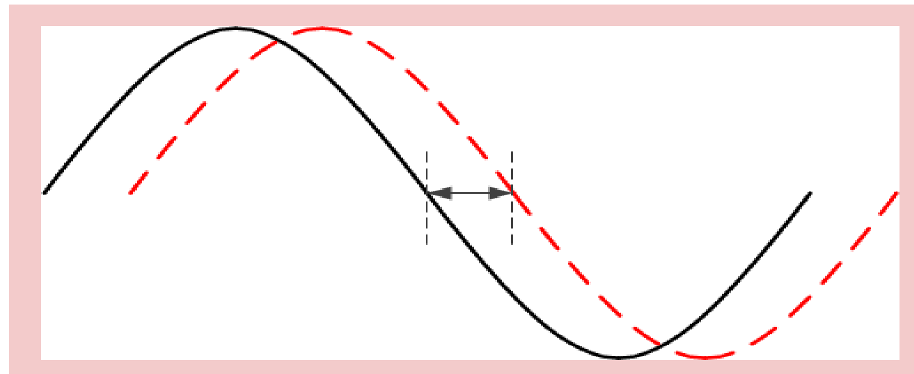


Types of Digital Modulation

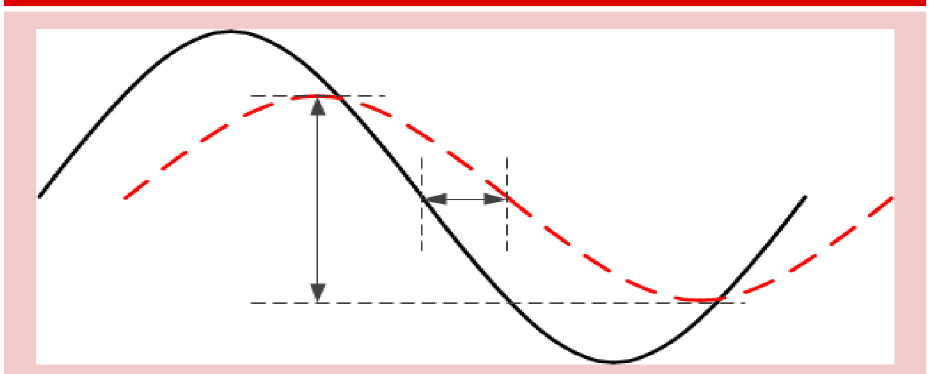
Pulse Amplitude Modulation (PAM)



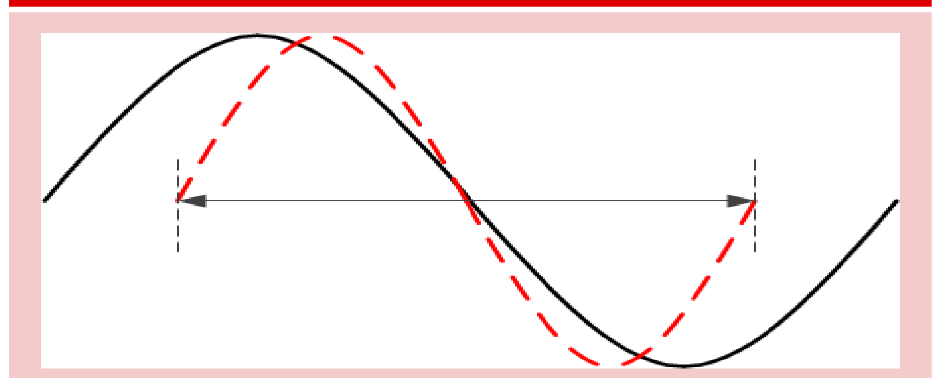
Phase Shift Keying (PSK)



Quadrature Amplitude Modulation (QAM)



Frequency Modulation (FM)



Phase and Amplitude Modulation

- A phase and amplitude modulated carrier signal can be represented as:

$$\boxed{X(t) = a(t)\cos\big(2\pi f_c t + \theta(t)\big)}$$

Where:

$a(t)$ is the amplitude modulation function

$\theta(t)$ is the phase modulation function

f_c is the carrier frequency

- Which can be decomposed using the angle sum trigonometric identity into:

$$X(t) = a(t)\cos[\theta(t)]\cos(2\pi f_c t) - a(t)\sin[\theta(t)]\sin(2\pi f_c t)$$

$$X(t) = X_I(t)\cos(2\pi f_c t) - X_Q(t)\sin(2\pi f_c t)$$

This is a “bandpass” signal since the I and Q functions are modulating a carrier

Where:

$X_I(t)$ is the “in-phase” modulation function

$X_Q(t)$ is the “quadrature-phase” modulation function

Often called “I” and “Q”

Analytical Equivalent and Baseband Equivalent

- The “bandpass” signal can be decomposed into a “lowpass” modulation function multiplied by a complex exponential (carrier) by using Euler’s formula and defining a new “baseband equivalent” modulation function, $X_{bb}(t)$:

Recall Euler’s formula:

$$e^{jx} = \cos x + j \sin x$$

Baseband equivalent

$$\boxed{X_{bb}(t) = X_I(t) + jX_Q(t)}$$

Then:

$$X(t) = X_I(t)\cos(2\pi f_c t) - X_Q(t)\sin(2\pi f_c t) \boxed{=} \boxed{Real\{X_{bb}(t)e^{j2\pi f_c t}\}}$$

Analytic equivalent

Real Modulation Example (Real Mixing)

- As an example of real modulation, consider the case of mixing a low frequency sine wave to a higher frequency, such that $X_{bb}(t)$ is simply a cosine wave:

$$X_{bb}(t) = \cos(2\pi f_{IF} t) = \frac{e^{j2\pi f_{IF} t} + e^{-j2\pi f_{IF} t}}{2}$$

From Euler's

- The resulting modulated signal is:

$$X(t) = Real\left\{\left(\frac{e^{j2\pi f_{IF} t} + e^{-j2\pi f_{IF} t}}{2}\right) e^{j2\pi f_c t}\right\}$$

$$X(t) = Real\left\{\frac{1}{2} e^{j2\pi f_{IF} t} e^{j2\pi f_c t} + \frac{1}{2} e^{-j2\pi f_{IF} t} e^{j2\pi f_c t}\right\}$$

$$X(t) = Real\left\{\frac{1}{2} e^{j2\pi (f_c + f_{IF}) t} + \frac{1}{2} e^{j2\pi (f_c - f_{IF}) t}\right\}$$

Desired signal

$$X(t) = \frac{1}{2}\cos(2\pi(f_c + f_{IF})t) + \frac{1}{2}\cos(2\pi(f_c - f_{IF})t)$$

Undesired Image signal

Complex Modulation Example (Complex Mixing)

- Define $X_{bb}(t)$ to be a complex exponential at the same frequency as the cosine in the previous example:

$$\boxed{X_{bb}(t) = e^{j2\pi f_{IF}t}} = \boxed{[\cos(2\pi f_{IF}t)] + j[\sin(2\pi f_{IF}t)]}$$

$X_I(t)$ $X_Q(t)$

- The resulting modulated signal is:

$$X(t) = Real\{e^{j2\pi f_{IF}t}e^{j2\pi f_c t}\} = \boxed{Real\{e^{j2\pi(f_c+f_{IF})t}\}}$$
$$X(t) = Real\{\cos(2\pi(f_c + f_{IF})t) + \boxed{j\sin(2\pi(f_c + f_{IF})t)}\}$$

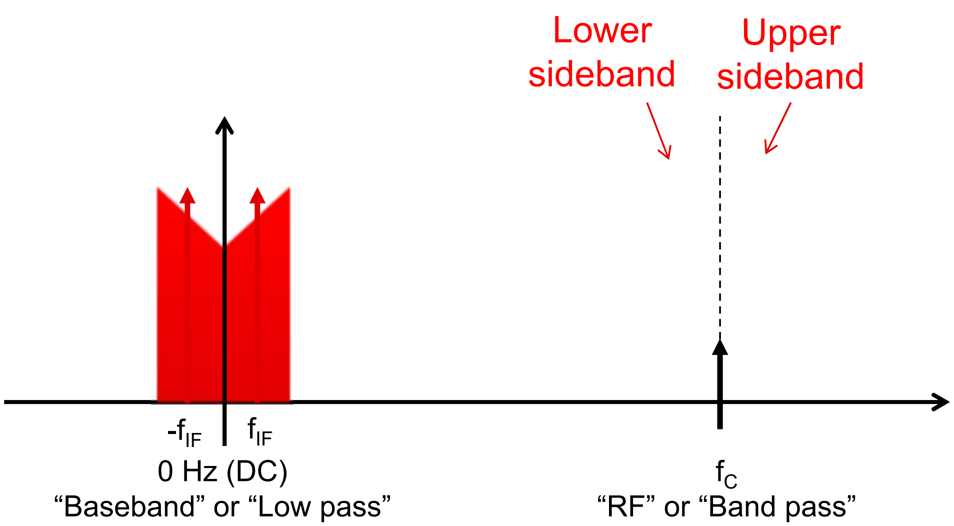
From Euler's

$$X(t) = \boxed{\cos(2\pi(f_c + f_{IF})t)}$$

Desired signal only

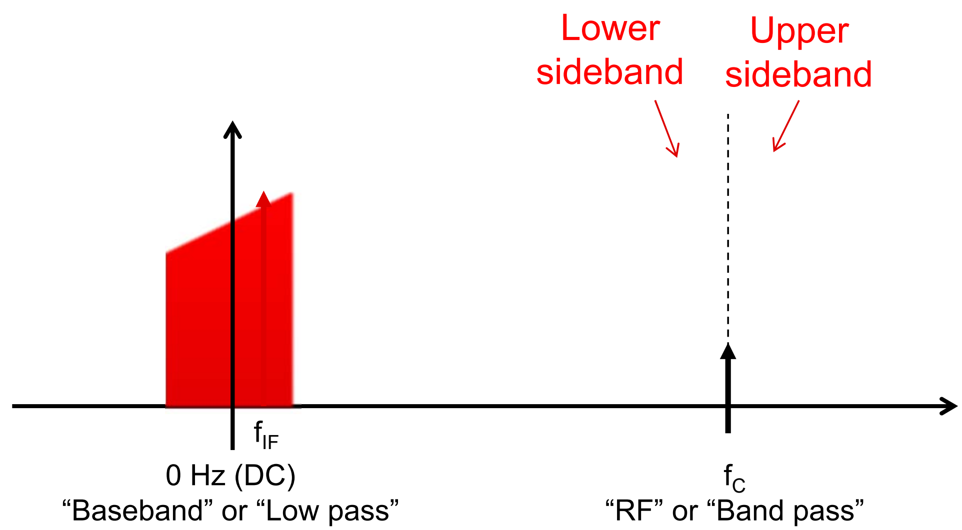
Real vs Complex Example Visualized

Real signals have equivalent (mirrored) positive and negative frequency spectrums:



This type of transmission is called "double sideband" (DSB) transmission since the same information is transmitted in both sidebands

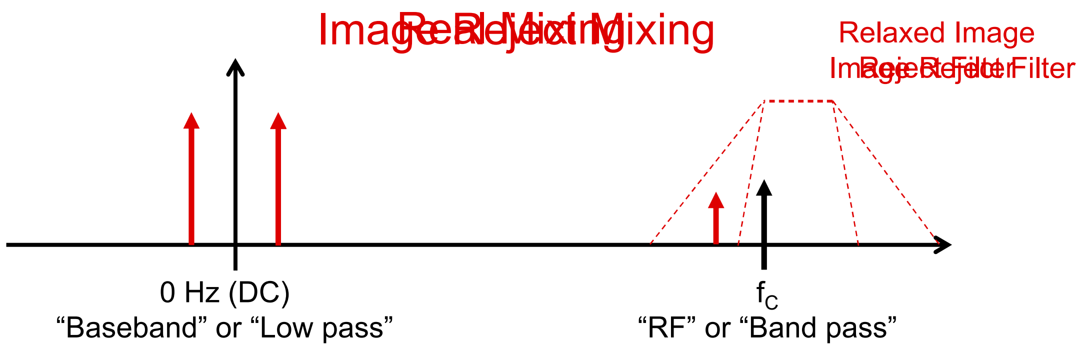
Complex signals have independent positive and negative frequency spectrums:



This type of transmission is called "single sideband" (SSB) transmission since each sideband can transmit unique information

Applications of Complex Modulation

- Image reject mixing uses the concept of complex modulation, exactly as shown in the complex modulation example, to reject the image signal in order to relax filtering requirements



- Digital communications uses complex modulation to “double” the data rate for a given signal bandwidth, ultimately using sine and cosine as an orthogonal basis in order to transmit independently on the in-phase and quadrature-phase signals

$$X(t) = X_I(t)\cos(2\pi f_c t) - X_Q(t)\sin(2\pi f_c t)$$

Transmit independent information on both I and Q

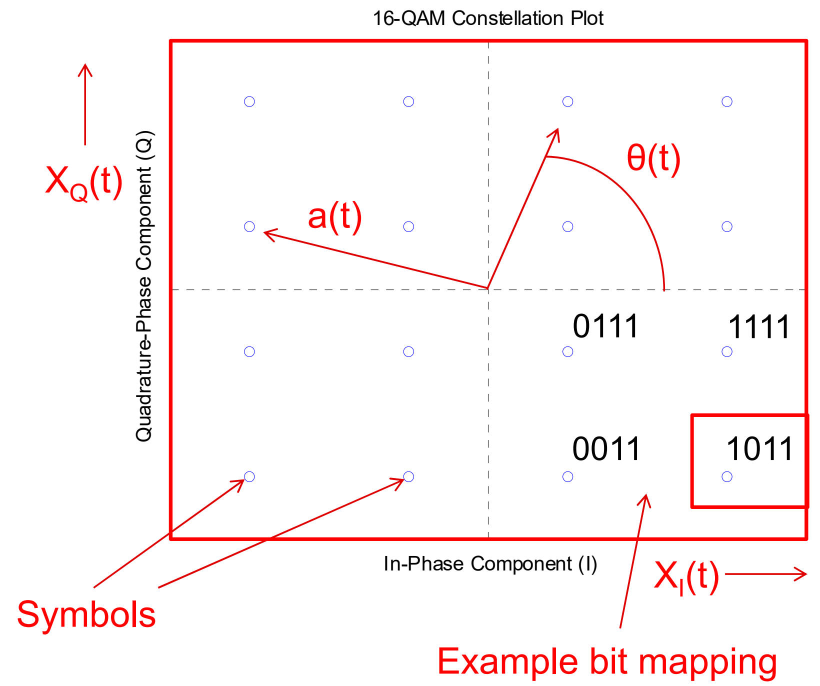
Digital Communications and Constellation Plots

- Constellation plots are used to visualize the complex baseband modulation function, $X_{bb}(t)$, by mapping the complex values onto the complex plane
- Amplitude of the carrier is the distance from the origin:

$$\boxed{a(t) = \sqrt{X_I^2(t) + X_Q^2(t)}}$$

- Phase of the carrier is the angle measured relative to the positive I axis:

$$\boxed{\theta(t) = \tan^{-1}\frac{X_Q(t)}{X_I(t)}}$$



Find Transmitted Signal from Constellation Plot

- We want to transmit 0011 across the channel using the 16-QAM plot on the previous slide
- To send 0011, choose I = 1, Q = -3:

$$\boxed{X_{bb}(t) = X_I(t) + jX_Q(t) = 1 - j3}$$

- Carrier amplitude:

$$\boxed{a(t) = \sqrt{X_I^2(t) + X_Q^2(t)} = \sqrt{1^2 + (-3)^2} = \sqrt{10}}$$

- Carrier phase:

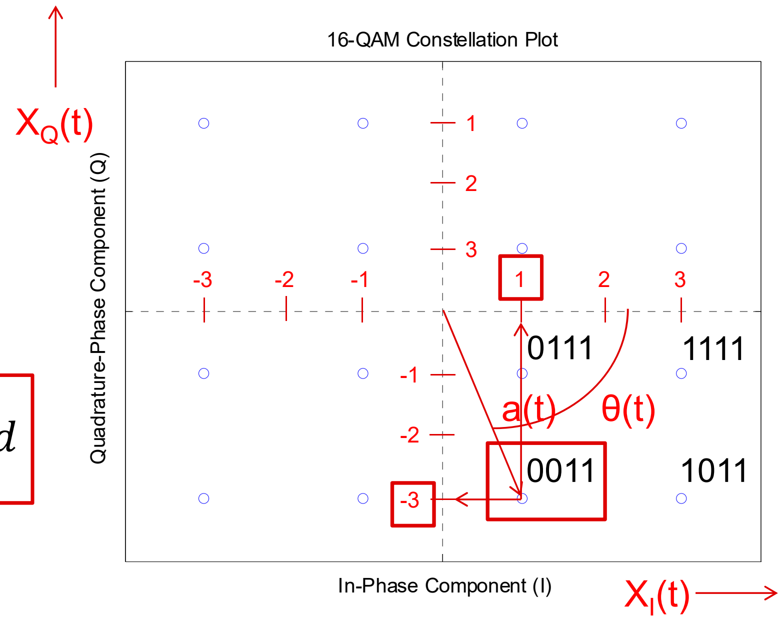
$$\boxed{\theta(t) = \tan^{-1}\frac{X_Q(t)}{X_I(t)} = \tan^{-1}\frac{-3}{1} = -71.6° = -1.25\,rad}$$

- Resulting signal, X(t):

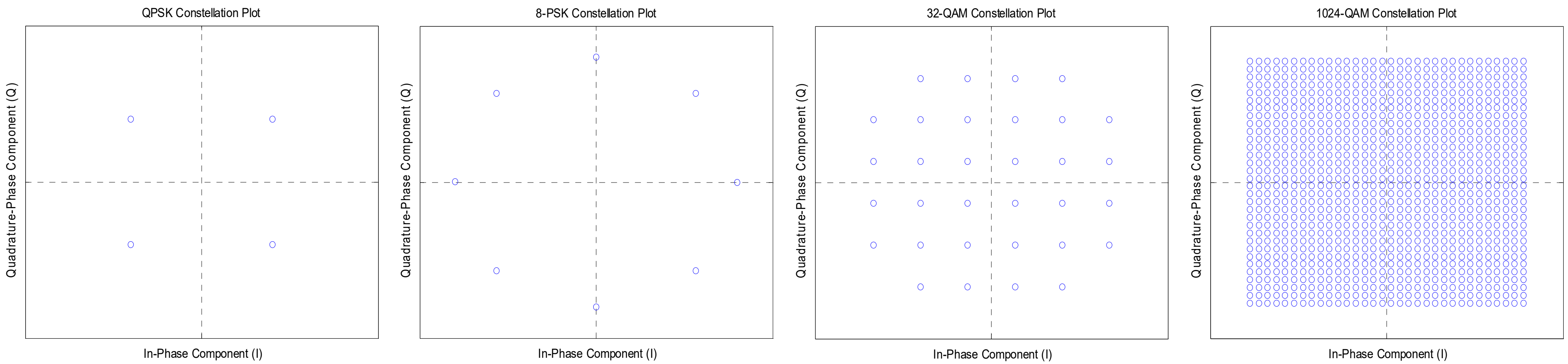
$$\boxed{X(t) = \sqrt{10}\cos(2\pi f_c t - 1.25)}$$

$$\boxed{X(t) = \cos(2\pi f_c t) - 3\sin(2\pi f_c t)}$$

Easier to implement in digital hardware



Example Constellation Plots



More challenging noise and distortion requirements →

← Lower data rate (fewer bits per symbol)

Thanks for your time!